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Question Paper Code : 20376

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Seventh/Eighth Semester

Computer Science and Engineering

CS 6702 — GRAPH THEORY AND APPLICATIONS

(Common to Information Technology)

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. For which of the following does there exist a simple graph $G=(V, E)$ satisfying the specified conditions?

   (a) It has 3 components 20 vertices and 16 edges

   (b) It is connected and has 10 edges 5 vertices and fewer than 6 cycles.

   (c) It has 7 vertices, 10 edges and more than two components.

2. The maximum degree of any vertex in a simple graph with n vertices is $n-1$. Give reasons.

3. Calculate the maximum flow between the nodes A and F in the graph Fig 3. The number on a line represents the capacity.

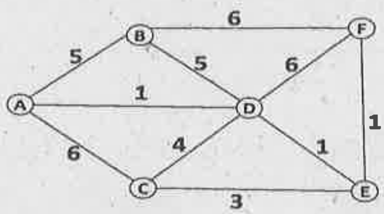


Figure – 3

4. Define Homeomorphic graphs and give example graphs.

5. What is meant by independent set and maximal independent set of a graph?

6. What is meant by regularization of a planar graph? Give an example.

7. How many ways can the letters in ENGINEERING be arranged so that all three E's together?

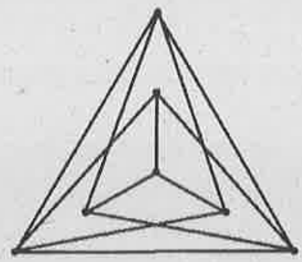
8. Determine the number of positive integers n, $1 \leq n \leq 500$, that are not divisible by 5 or 5.

9. Define generating function. Give an example to a polynomial and a power series.

10. Write a homogeneous and a non-homogeneous recurrence relations.

PART B — (5 × 13 = 65 marks)

11. (a) (i) Draw a graph isomorphic to the graph G shown in Figure 11(a) (i) such that no edge is crossing others. (5)



G

Figure – 11 (a) (i)

(ii) Define walk, circuit, path and subgraph. From the graph shown in figure 11(a)(ii), draw a walk of any length, a path of length 5, a circuit of length 4 and subgraph of 4 vertices and 5 edges. (8)

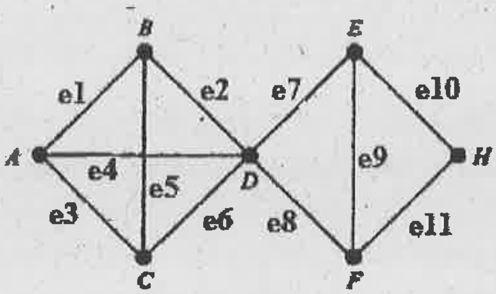


Figure – 11 (a) (ii)

Or

(b) (i) Seven children in a street play a game in a circular arrangement. If no child holds hands with the same playmate twice, how many times can this arrangement possible? Write all possible arrangements. (5)

(ii) Prove that there are at least two pendant vertices in a tree with two or more vertices. Also prove that every tree has one or two centers. (8)

12. (a) (i) Prove that the distance between any two spanning trees is a metric. Find two different minimum spanning trees of a graph with $V=\{1,2,3,4\}$ is described by

$$\varphi=\begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{1,2\} & \{1,4\} & \{2,3\} & \{3,4\} & \{3,4\} \end{pmatrix}.$$ It has weights on its edges given by $\lambda=\begin{pmatrix} a & b & c & d & e & f \\ 3 & 2 & 1 & 2 & 4 & 2 \end{pmatrix}$. (7)

(ii) Prove that an Euler graph cannot have a cut-set with an odd number of edges. (6)

Or

(b) (i) Construct a graph G with the following properties: Edge connectivity of G = 4, vertex connectivity of G = 3, and degree of every vertex of $G \geq 5$. (7)

(ii) Derive the formula for the number of regions in a planar graph, G with n vertices and e edges. Also prove that a planar graph with triangle regions can have at most $(3n-6)$ edges. (6)

13. (a) (i) Define chromatic polynomial and write the chromatic polynomial of a graph with n vertices. (5)

(ii) Define complete matching and minimal covering in a graph G. Give one application example to each. (8)

Or

(b) (i) Define the following and give one example to each.

(1) Complete symmetric digraph

(2) Balanced digraph

(3) Equivalence graph

(4) Accessibility in a digraph. (8)

(ii) When is a digraph an Euler digraph? Draw an Euler digraph. (5)

14. (a) Determine the number of six digit integers (no leading zeros) in which

(i) No digit may be repeated;

(ii) Digits may be repeated. Answer parts

(i) and (ii) with the extra condition that the six digit number even; Also do the same with the condition that the number is divisible by 5. (13)

Or

(b) At a nursery, Reshmi wants to arrange 15 different plants on five shelves for a window display. In how many ways can she arrange them so that each shelf has at least one, but no more than four plants? (13)

15. (a) (i) What is Ferrer's graph? Give an example Ferrer's graph and its transposition graph. (5)

(ii) Explain exponential generating function with an example. (8)

Or

(b) (i) Explain the summation operator with an example. (5)

(ii) What is meant by a recurrence relation? Write one applications of each first order and second order linear homogeneous recurrence relation with examples. (8)

PART C — (1 × 15 = 15 marks)

16. (a) Can the kolams shown in figure 16 (a) (i) and (ii) be drawn without lifting your hands and not overdrawing any part of the kolam? Substantiate your answers with graph theory knowledge. If not possible, make it possible by adding some curves. (8+7)

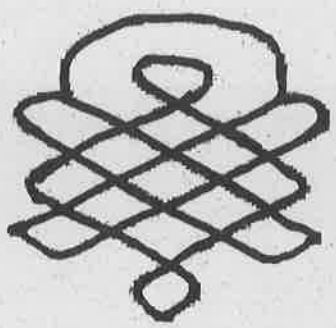


Figure – 16 (a) (i)

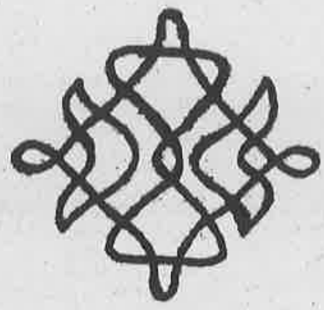


Figure – 16 (a) (ii)

Or

(b) (i) Stack the blocks shown in figure 16 (b) (i) in a pile of 4 in such a way that each of the colors appears exactly once on each of the four sides of the stack. (7)

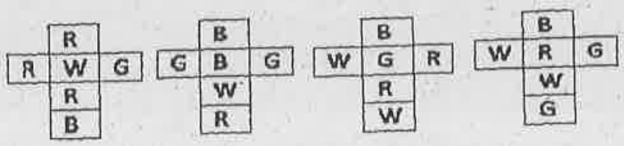


Figure – 16 (b) (i)

(ii) A young pair of rabbits, one of each sex, is placed on an island. A pair of rabbits does not breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month. Assume that none of the rabbits die. How many rabbits are there after n months? (8)